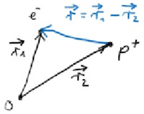
2 body



$$\hat{H} = \frac{\hat{p}_1^2}{2m_1} + \frac{\hat{p}_2^2}{2m_2} + U(\underline{r}_1 - \underline{r}_2)$$

$$\vec{p} = -i\hbar\vec{\nabla}$$

$$\Rightarrow \hat{H} = -\frac{\hbar^2}{2m_1}\Delta_1 - \frac{\hbar^2}{2m_2}\Delta_2 + U(\underline{r}_1 - \underline{r}_2)$$

$$(\underline{r}_1, \underline{r}_2) \Rightarrow (\underline{R}_{CM}, \underline{r})$$

$$\left.\begin{aligned} \underline{R}_{CM} &= \frac{m_1\underline{r}_1 + m_2\underline{r}_2}{m_1+m_2} \\ \underline{r} &= \underline{r}_1 - \underline{r}_2 \end{aligned}\right\} \Rightarrow \begin{aligned} \underline{r}_1 &= \underline{R}_{CM} + \frac{m_2}{m_1+m_2}\cdot\underline{r} \\ \underline{r}_2 &= \underline{R}_{CM} - \frac{m_1}{m_1+m_2}\cdot\underline{r} \end{aligned}$$

$$f(\underline{r}_1, \underline{r}_2) \equiv f\left(\underline{R}_{CM}(\underline{r}_1,\underline{r}_2), \underline{r}(\underline{r}_1,\underline{r}_2)\right)$$

$$\text{"}\frac{\partial \underline{R}_{CM}}{\partial \underline{r}_i} = \frac{m_i}{m_1+m_2}\text{"} \qquad \text{"}\frac{\partial \underline{r}}{\partial \underline{r}_1} = 1\text{"} \qquad \text{"}\frac{\partial \underline{r}}{\partial \underline{r}_2} = -1\text{"}$$

$$! \quad m_1 + m_2 = M$$

$$\Rightarrow \frac{\partial}{\partial x_1} = \frac{\partial}{\partial x_{CM}}\cdot\frac{m_1}{M} + \frac{\partial}{\partial x}\cdot 1$$

$$\frac{\partial}{\partial x_2} = \frac{\partial}{\partial x_{CM}}\cdot\frac{m_2}{M} + \frac{\partial}{\partial x}\cdot(-1)$$

$$\Rightarrow \frac{\partial^2}{\partial x_1^2} = \left(\frac{m_1}{M}\frac{\partial}{\partial x_{CM}} + \frac{\partial}{\partial x}\right)\left(\frac{m_1}{M}\frac{\partial}{\partial x_{CM}} + \frac{\partial}{\partial x}\right) = \frac{m_1^2}{M^2}\frac{\partial^2}{\partial x_{CM}^2} + \frac{\partial^2}{\partial x^2} + \cancel{2\cdot\frac{m_1}{M}\frac{\partial^2}{\partial x\partial x_{CM}}}$$

$$\frac{\partial^2}{\partial x_2^2} = \left(\frac{m_2}{M}\frac{\partial}{\partial x_{CM}} - \frac{\partial}{\partial x}\right)\left(\frac{m_2}{M}\frac{\partial}{\partial x_{CM}} - \frac{\partial}{\partial x}\right) = \frac{m_2^2}{M^2}\frac{\partial^2}{\partial x_{CM}^2} + \frac{\partial^2}{\partial x^2} - \cancel{2\cdot\frac{m_2}{M}\frac{\partial^2}{\partial x\partial x_{CM}}}$$

$$\Rightarrow \frac{1}{m_1}\frac{\partial^2}{\partial x_1^2} + \frac{1}{m_2}\frac{\partial^2}{\partial x_2^2} = \dots = \underbrace{\frac{m_1+m_2}{M^2}}_{1/M}\frac{\partial^2}{\partial x_{CM}^2} + \underbrace{\left(\frac{1}{m_1}+\frac{1}{m_2}\right)}_{1/m_{red}}\frac{\partial^2}{\partial x^2} + \emptyset$$

$$\Rightarrow \hat{H} = -\frac{\hbar^2}{2m_1}\Delta_1 - \frac{\hbar^2}{2m_2}\Delta_2 + U(\underline{r}_1 - \underline{r}_2) = \underbrace{-\frac{\hbar^2}{2M}\Delta_{CM}}_{\hat{H}_{CM}}\underbrace{-\frac{\hbar^2}{2m_{red}}\cdot\Delta_{rel} + U(\underline{r}_{rel})}_{\hat{H}_{rel}}$$

$$\hat{H}\psi = i\hbar\frac{\partial\psi}{\partial t}$$

$$\hat{H} = \hat{H}_{CM} + \hat{H}_{rel} \quad \Rightarrow \psi(\underline{R}_{CM}, \underline{r}_{rel}) = \psi_{CM}(\underline{R}_{CM})\cdot\psi_r(\underline{r}_{rel})$$

$$\Rightarrow (\hat{H}_{CM}\psi_{CM})\cdot\underline{\psi_r} + (\hat{H}_{rel}\psi_r)\cdot\underline{\underline{\psi_{CM}}} = i\hbar\cdot\dot{\psi}_{CM}\cdot\underline{\psi_r} + i\hbar\,\dot{\psi}_r\cdot\underline{\underline{\psi_{CM}}}$$

$$\Rightarrow (\hat{H}_{CM}\psi_{CM} - i\hbar\dot{\psi}_{CM})\cdot\psi_r + (\hat{H}_{rel}\psi_r - i\hbar\dot{\psi}_r)\cdot\psi_{CM} = 0 \;\Big/ \cdot \text{"}\frac{1}{\psi_{CM}\cdot\psi_r}\text{"}$$

$$\Rightarrow \begin{aligned} \hat{H}_{CM}\psi_{CM} &= i\hbar\,\dot{\psi}_{CM} && \Rightarrow \text{plane wave} \\ \hat{H}_{rel}\psi_r &= i\hbar\,\dot{\psi}_r && \Rightarrow \text{bound state (H-atom)} \end{aligned}$$

$$m_{el} \to m_{red} = \frac{m_{el}\cdot m_{nuc}}{m_{el}+m_{nuc}} = m_{el}\cdot\frac{1}{1+\frac{m_{el}}{m_{nuc}}} \approx m_{el}\cdot\left(1 - \underbrace{\frac{m_{el}}{m_{nuc}}}_{\lesssim 5\cdot 10^{-4}}\right)$$