

$$E(\psi_0 + \delta\psi) \approx E(\psi_0) + \frac{\delta E}{\delta\psi} + \mathcal{O}(\delta\psi^2)$$

\uparrow
 opt. δ (dipol moment) $\sim \delta\psi$

Eckart inequality

$$|\psi\rangle = \sum c_i |\psi_i\rangle \quad \langle\psi|\psi\rangle = 1 \quad \sum_{i=0}^{\infty} |c_i|^2 = 1$$

$$S = \langle\psi_0|\psi\rangle$$

$$E = \langle\psi|H|\psi\rangle = \dots = \sum_{i=0}^{\infty} |c_i|^2 \cdot E_i = |S|^2 \cdot E_0 + \sum_{i=1}^{\infty} |c_i|^2 \cdot E_i \geq E_1$$

$$\geq |S|^2 \cdot E_0 + \sum_{i=1}^{\infty} |c_i|^2 \cdot E_1 = |S|^2 \cdot E_0 + E_1 \cdot \underbrace{\sum_{i=1}^{\infty} |c_i|^2}_{1 - |S|^2} =$$

$$= |S|^2 \cdot E_0 + E_1 (1 - |S|^2)$$

$$(E_1 - E_0) \cdot |S|^2 \geq E_1 - E$$

$$1 \geq |S|^2 \geq \frac{E_1 - E}{E_1 - E_0}$$

$\rightarrow 1$

